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$$\therefore -\frac{4\theta^2}{\pi^2}\left(1+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\right)-\frac{4^2\theta^4}{2}\left(1+\frac{1}{3^4}+\frac{1}{5^4}+\ldots\ldots\right)-\ldots\ldots$$

$$-\frac{4^8\theta^{16}}{8\pi^{16}}\left(1+\frac{1}{3^{16}}+\frac{1}{5^{16}}+\frac{1}{7^{16}}\right)=-A-\tfrac{1}{2}A^2-\ldots\ldots-\tfrac{1}{8}A^8-\ldots\ldots$$

Equating like coefficients of θ^{16} we get

$$\frac{8192}{\pi^{16}}\left(1+\frac{1}{3^{16}}+\frac{1}{5^{16}}+\frac{1}{7^{16}}+\ldots\ldots\right)=\frac{929569}{6435\times1260^2}.$$

$$\therefore \frac{6435}{2}\cdot\frac{161280^2}{929569}\left(1+\frac{1}{3^{16}}+\frac{1}{5^{16}}+\frac{1}{7^{16}}+\ldots\ldots\right)=\pi^{16}.$$

Also solved by F. Anderegg, and G. W. Greenwood.

249. Proposed by J. J. KEYES, Fogg High School, Nashville, Tenn.

Solve $x+y+z=5$, $x^2+y^2=z^2$, $x^3+y^3+z^3=8$.

Solution by M. R. BECK, Cleveland, Ohio.

$x+y+z=5$, or $x+y=5-z$.........(1), $x^2+y^2=z^2$.........(2),
$x^3+y^3+z^3=8$, or $(x+y)(x^2-xy+y^2)=8-z^3$.........(3).

From (1) and (2), we have $xy=\dfrac{25-10z}{2}$.........(4).

Substituting (1), (2), and (4) in (3), and solving, $z=\frac{47}{25}$. Substituting $z=\frac{47}{25}$ in (1) and (2) and solving, $x=\frac{39}{25}\mp\frac{7}{50}\sqrt{-34}$, and $y=\frac{39}{25}\pm\frac{7}{50}\sqrt{-34}$.

Also solved by Henry Heaton, A. H. Holmes, L. E. Newcomb, J. Scheffer, and G. B. M. Zerr.

AVERAGE AND PROBABILITY.

172. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

What is the average length of all straight lines that can be drawn within a given triangle?

II. Solution by HENRY HEATON, Atlantic, Iowa.

Let ABC be the triangle. Let x=length of one of the straight lines, and θ the angle it would make with the side AB if produced to meet it, θ being taken $>$ than the angle A and $<$ than $\pi-B$. At the distance $x\sin\theta$ from AB draw ED parallel to AB cutting AC in E and BC in D. Then the number of lines of length x making the angle θ with AB is equal to the number of points in the triangle DEC whose area is $\dfrac{\Delta\,(b\sin A-x\sin\theta)^2}{b^2\sin^2A}$. Similarly, the number of lines of length x making the angle θ with AC is equal to the number of points in the triangle whose area is $\dfrac{\Delta\,(c\sin A-x\sin\theta)^2}{c^2\sin^2A}$, and the number making same angle with the side BC is equal to the number of points in the triangle whose area is

$\frac{\triangle\ (c\sin B-x\sin\theta)^2}{c^2\sin^2 B}$. Then if all lines are equally distributed about the starting point the required average is

$$M_1=\frac{\int_A^{\pi-B}\int_0^{\frac{b\sin A}{\sin\theta}}\frac{(b\sin A-x\sin\theta)^2}{b^2\sin^2 A}d\theta x dx+\int_A^{\pi-C}\int_0^{\frac{c\sin A}{\sin\theta}}\frac{(c\sin A-x\sin\theta)^2}{c^2\sin^2 A}d\theta x dx}{\int_A^{\pi-B}\int_0^{\frac{b\sin A}{\sin\theta}}\frac{(b\sin A-x\sin\theta)^2}{b^2\sin^2 A}d\theta\, dx+\int_A^{\pi-C}\int_0^{\frac{c\sin A}{\sin\theta}}\frac{(c\sin A-x\sin\theta)^2}{c^2\sin^2 A}d\theta dx}$$

$$\frac{+\int_B^{\pi-C}\int_0^{\frac{c\sin B}{\sin\theta}}\frac{(c\sin B-x\sin\theta)^2}{c^2\sin^2 B}d\theta x dx}{+\int_B^{\pi-C}\int_0^{\frac{c\sin B}{\sin\theta}}\frac{(c\sin B-x\sin\theta)^2}{c^2\sin^2 B}d\theta dx}$$

$$=\frac{\int_A^{\pi-B}\frac{b^2\sin^2 A}{\sin^2\theta}d\theta+\int_A^{\pi-C}\frac{c^2\sin^2 A}{\sin^2\theta}d\theta+\int_B^{\pi-C}\frac{c^2\sin^2 B}{\sin^2\theta}d\theta}{4\int_A^{\pi-B}\frac{b\sin A}{\sin\theta}d\theta+4\int_A^{\pi-C}\frac{c\sin A}{\sin\theta}d\theta+4\int_B^{\pi-C}\frac{c\sin B}{\sin\theta}d\theta}$$

$$=\tfrac{3}{4}abc\div ab\log\left(\frac{s}{s-c}\right)+ac\log\left(\frac{s}{s-b}\right)+bc\log\left(\frac{s}{s-a}\right).$$

If the lines are so distributed as to join every possible pair of points the required average is

$$M_2=\frac{\int_A^{\pi-B}\int_0^{\frac{b\sin A}{\sin\theta}}\frac{(b\sin A-x\sin\theta)^2}{b^2\sin^2\theta}d\theta x^2 dx+\int_A^{\pi-C}\int_0^{\frac{c\sin A}{\sin\theta}}\frac{(c\sin A-x\sin\theta)^2}{c^2\sin^2\theta}d\theta x^2 dx}{\int_A^{\pi-B}\int_0^{\frac{b\sin A}{\sin\theta}}\frac{(b\sin A-x\sin\theta)^2}{b^2\sin^2 A}d\theta x dx+\int_A^{\pi-C}\int_0^{\frac{c\sin A}{\sin\theta}}\frac{(c\sin A-x\sin\theta)^2}{c^2\sin^2 A}d\theta x dx}$$

$$\frac{+\int_B^{\pi-C}\int_0^{\frac{c\sin B}{\sin\theta}}\frac{(c\sin B-x\sin\theta)^2}{c^2\sin^2 B}d\theta\, x^2 dx}{+\int_B^{\pi-C}\int_0^{\frac{c\sin B}{\sin\theta}}\frac{(c\sin B-x\sin\theta)^2}{c^2\sin^2 B}d\theta x dx}$$

$$=\frac{2}{5}\frac{\left[\int_A^{\pi-B}\frac{b^3\sin^3 A}{\sin^3\theta}d\theta+\int_A^{\pi-C}\frac{c^3\sin^3 A}{\sin^3\theta}d\theta+\int_B^{\pi-C}\frac{c^3\sin^3 B}{\sin^3\theta}d\theta\right]}{\int_A^{\pi-B}\frac{b^2\sin^2 A}{\sin^2\theta}d\theta+\int_A^{\pi-C}\frac{c^2\sin^2 A}{\sin^2\theta}d\theta+\int_B^{\pi-C}\frac{c^2\sin^2 B}{\sin^2\theta}d\theta}$$

$$=\tfrac{1}{15}\left(a+b+c + (a+b)\frac{(a-b)^2}{c^2}+(a+c)\ \frac{(a-c)^2}{b^2}+ (b+c)\,\frac{(b-c)^2}{a^2}\right)$$

$$+\tfrac{4}{15}\,\triangle^2\left[\frac{1}{c^3}\log\left(\frac{s}{s-c}\right)+\frac{1}{b^3}\log\left(\frac{s}{s-b}\right)+\frac{1}{a^3}\log\left(\frac{s}{s-a}\right)\right].$$

In the above free use is made of the well known formulas

$$\sin A=\frac{2\ \triangle}{bc},\quad \cos A=\frac{b^2+c^2-a^2}{bc},\quad \tan\tfrac{1}{2}A=\sqrt{\frac{(s-b)\ (s-c)}{s(s-a)}},\ \text{etc.}$$

*If $c=b=a$, $M_1=\dfrac{a}{4\log 3}=.2276a$, and $M_2=\frac{1}{5}+\frac{4}{5}\dfrac{\triangle^2}{a^3}\log 3=\dfrac{a}{5}\,(1+\frac{3}{4}\log 3)$ $=.3638a$.

CALCULUS.

Note on Problem 207.—The proposer's value of $\frac{1}{4}\pi^2$ for the definite integral $\displaystyle\int_0^1\frac{Kd_\kappa}{1+\kappa}$ is wrong and should be $\frac{1}{8}\pi^2$. Mr. Corey's solution (p. 230) is correct except in the last line where the evaluation of $u-v=2\displaystyle\int_0^1\frac{\log(1+z)}{z}dz-\int_0^1\frac{\log(1+z^2)}{z}$ should read $2\dfrac{\pi^2}{12}-\frac{1}{2}\dfrac{\pi^2}{12}=\dfrac{\pi^2}{8}$. That this is true is easily shown by the following evaluation, due to Dr. Zerr, who has contributed two correct solutions of the problem.

$$u-v=2\int_0^1(1-\tfrac{1}{2}z+\tfrac{1}{3}z^2-\tfrac{1}{4}z^3+\ldots\ldots)dz-\int_0^1(z-\tfrac{1}{2}z^3+\tfrac{1}{3}z^5-\tfrac{1}{4}z^7+\ldots\ldots)dz$$

$$=2(1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots)-\tfrac{1}{2}(1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots)$$

$$=-\tfrac{3}{2}(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\ldots-\frac{2}{1^2}-\frac{2}{3^2}-\frac{2}{5^2}-\ldots\ldots)\dagger=-\tfrac{3}{2}(\frac{\pi^2}{6}-\frac{\pi^2}{4})=\frac{\pi^2}{8}.$$

G.

210. Proposed by EDWIN L. RICH, Schenectady, New York.

Determine a polynomial, $f(x)$, entirely in x and of the seventh degree, so that $f(x)+1$ is divisible by $(x-1)^4$ and $f(x)-1$ by $(x+1)^4$. [Goursat-Hedrick, *A Course in Mathematical Analysis*, p. 32, Ex. 3.]

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*See Problem 35, p. 391, Williamson's *Integral Calculus*.

†See for example, Locke's *Treatise on Higher Trigonometry*, p. 99 and Ex. 1, p. 100.